Technical Comments

Comment on "Dynamic Equations for Connected Rigid Bodies"

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A DYNAMICAL formalism for systems of rigid bodies, with the needs of digital simulation especially in mind, has recently been given by Sandler.[1] The philosophy he outlines is quite correct, though some elaboration of certain definitions might be desirable. I wish to comment that there already is a body of literature on this subject, including Refs. 2–6, and some of these carry the development further. (I do not cite the other formalisms of the same general type given in various unpublished works, i.e., company reports, during the past decade.) References 5 and 6, in particular, also were developed with a recognition of digital simulation problems.

I feel that Sandler's concluding remarks about "elastic bodies" are subject to misinterpretation, since this terminology usually is interpreted to refer to elastic continua to which his formulation (as well as the others cited here) does not apply.

References

[1] Sandler, S. H., "Dynamic equations for connected rigid bodies," J. Spacecraft Rockets **4,** 684–685 (1967).

[2] Roberson, R. E., "Attitude control of satellites and space vehicles," *Advances in Space Science* (Academic Press Inc., New York, 1960), Vol 2, pp. 351–436.

[3] Grubin, C., "Dynamics of a vehicle containing moving parts," J. Appl. Mech. **29,** 486–488 (1962).

[4] Abzug, M. J., "Active satellite attitude control," *Guidance and Control of Aerospace Vehicles*, edited by C. T. Leondes (McGraw-Hill Book Company Inc., New York, 1963), pp. 331–425; also "Attitude control of multiple-part satellites," Ph.D. dissertation, Univ. of California at Los Angeles (January 1962).

[5] Hooker, W. W. and Margulies, G., "The dynamical attitude equations for an n-body satellite," J. Astronaut. Sci. **12,** 123–128 (1965).

[6] Roberson, R. E. and Wittenburg, J., "A dynamical formalism for an arbitrary number of interconnected rigid bodies, with reference to the problem of satellite attitude control," *Proceedings of the Third Congress International Federation of Automatic Control* (Butterworths Scientific Publications Ltd., London, to be published).

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Reply by Author to R. E. Roberson

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I WOULD like to thank Professor Roberson for providing a list of very worthwhile references on the subject matter of my note.[1] I became aware of some of these in addition to Ref. 2 after submitting the note for final printing.

It was hoped that the discussion about elastic bodies would not lead to confusion since it was stated "...if it can be assumed that the body consists of lumped masses and massless springs." Modelling of elastic bodies by idealized deformable bodies such as massless springs has been very effective in structural dynamic analysis.

The analyses of Roberson's Refs. 5 and 6 are sufficiently general for application to certain classes of elastic bodies, namely those that can be modelled by rigid bodies connected by massless rotational springs. The discussion of elastic bodies in Ref. 1 was only intended to indicate that such analyses can be further generalized to include translational springs between rigid bodies. Unfortunately, the discussion appears to be inadequate.

A report[3] has been written which generalizes the analyses of Roberson's Refs. 5 and 6 to include idealized deformable bodies. The equations in the report are also applicable to bodies connected in a topological mesh configuration. The previous analyses have been restricted to topological tree configurations.

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References

[1] Sandler, S. H., "Dynamic equations for connected rigid bodies," J. Spacecraft Rockets **4,** 684–685 (1967).

[2] Chobotov, V., "An extension of the satellite attitude determination and control digital simulation," Aerospace Corp., ATM-66(6532)-3 (August 1965).

[3] Sandler, S. H. and Worrell, E. A., "General equations of motion for an arbitrary satellite," Communication Systems Inc. Report (to be released).

Comment on "Jet Compression for Closed-Cycle Magnetoplasmadynamic Electrical Power Generation"

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Nomenclature

A	= cross-sectional flow area, m^2
C	= $\rho_3 u_3$, Eq. (7), kg$_f$-sec/m^3
c_p	= specific heat at constant pressure, m^2/sec^2-°K
h	= specific enthalpy, m^2/sec^2
M	= Mach number
$\mathfrak{M}$	= molecular mass
$\dot{m}$	= mass flow rate, kg$_f$-sec/m
p	= pressure or partial pressure, kg$_f$/m^2
$\mathfrak{R}$	= universal gas constant, m^2/sec^2-°K
R	= $\mathfrak{R}/\mathfrak{M}$, specific gas constant, m^2/sec^2-°K
T	= absolute temperature, °K
u	= velocity, m/sec
x	= quality
α	= $\dot{m}_2/\dot{m}_1$, ratio of secondary to primary stream mass rate
Δh_v	= heat of vaporization per unit mass, m^2/sec^2
ρ	= density, kg$_f$-sec^2/m^4
$\varphi(T)$	= h_l, specific enthalpy of the liquid phase, m^2/sec^2
$\psi(T)$	= Δh_v, heat of vaporization per unit mass, m^2/sec^2
$X(T)$	= p_s, pressure of saturated vapor, kg$_f$/m^2

Subscripts

0	= related to stagnation conditions
1,2,3	= primary, secondary, and mixed stream, respectively

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